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DEVELOPING A TWO DIMENSIONAL CLIMATE RISK MODEL FOR DENGUE DISEASE TRANSMISSION IN URBAN COLOMBO

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AUTHORS' CONTRIBUTIONS

This work was carried out in collaboration between both authors. Authors WPTMW and SSNP designed the study, performed the simulation and data analysis. Author WPTMW wrote the first draft of the manuscript and managed literature searches. Author SSNP managed the editing of the manuscript. Both authors read and approved the final manuscript.

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Review Article

ABSTRACT

Dengue has been a major public health concern in the tropical world for decades now. The dynamics of dengue disease transmission are complex and uncertain due to various external factors such as climate, human behavior, geography and demography. Fuzzy logic and fuzzy set theory are useful tools in mathematics to model systems under uncertainty where classical approaches are insufficient. We are particularly concerned with modeling the potential risk of dengue disease transmission with respect to climate variables namely, rainfall and temperature. We define fuzzy membership functions for rainfall and temperature which describe the levels of unfavorable conditions to spread dengue. Then a modified version of the Einstein Sum Operator is used to measure the overall effect from rainfall and temperature; hence we obtain a fuzzy valued time series of potential risk produced by the climate. This risk measure is validated with the actual dengue cases reported in urban Colombo from year 2006 to 2015. The residuals of the predicted and the real risk of dengue transmission is less than 0.4 (80% accurate) in 86.77% of the time considered to the study. The sensitivity analysis of the model is also carried out to investigate how it responses to the measurement errors in the climate parameters.

Keywords: Dengue transmission; fuzzy operator; climate risk; sensitivity.

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1 INTRODUCTION

Dengue is an important public health problem in the world mainly in the tropical regions. It has been an epidemic disease in Sri Lanka now since it was identified five decades ago. Dynamics of dengue transmission have changed over the years mainly in the urban environment. It is an epidemic disease which is highly sensitive to climate and human behavior [1]. Some levels of rainfall support mosquito breeding and increased temperature levels reduce the incubation period of mosquitoes and increase the vector capacity to transmit the dengue virus [2].

Various researchers such as Puntani P. [3] and Lourdes E et al [4] have discussed the applicability of classical SIR models in terms of population dynamics to understand the dynamics of dengue disease transmission. The classical SIR models explain the interaction among of susceptible (S), infected (I) and recovered (R) human populations together with the susceptible and infected vector populations. However, they have used fixed parameter values in their models so that the influence of external factors to change the parameters has not been addressed adequately. The estimation of these parameters should be done under uncertainty and the fully stochastic models are not appropriate since we do not know the underlying probability distributions. Some researchers have attempted statistical models such as time series and Poisson Regression models but they only have predicted dengue in relation to changes in single external parameter such as rainfall, temperature, humidity and wind speed [5, 6]. Authors in [5] have studied spatio-temporal patterns of dengue transmission using long term climate data together with socio-ecological changes. Results in [6, 7] suggest the importance of temperature and precipitation in the transmission of dengue viruses and suggest a reason for their spatial heterogeneity. However, their models do not reflect an overall risk measure of dengue disease when all external parameters are taken together at different levels.

Fuzzy logic and fuzzy set theory are emerging areas in mathematical research which provide valuable and interesting input to model systems under uncertainty [8]. Fuzzy models in biology, epidemiology and medicine are having tremendous capabilities of explaining complex system behaviors under uncertainties. Plerou A. and et al have reviewed Fuzzy logic concepts and their applications to population biology with an emphasis on epidemiological problems like causal studies, epidemic models, and designing of vaccination strategies [9]. A study on including fuzzy parameters in epidemic modeling using fuzzy dynamical systems can be found in [10]. The results in [10] are interesting and they encourage to use fuzzy tools in modeling systems under uncertainty. There are some studies can be found discussing the applicability of fuzzy logic to Infectious Disease Diagnosis with the aid of Computer Science. Prihatini P.M. and et al develop an expert system combines the method of Fuzzy Logic and Certainty Factors with the object of research is a disease of tropical infectious diseases include Dengue Fever, Typhoid Fever and Chikungunya [11]. However, direct applications in modeling dengue disease transmission using fuzzy tools are not frequently availble in literature.

There are basically two objectives of this paper. The first aim is to model the influence of rainfall and temperature to create unfavorable environmental conditions for the spread of dengue disease in urban Colombo. We use fuzzy set theory to investigate the influence from eight weeks leading rainfall (RF) and the weekly averaged maximum temperature (TEMP) for dengue transmission [6]. The membership functions for weekly average rainfall with eight week lead time and weekly average maximum temperature are defined with the degree of membership value in [0, 1] as the response variable which is the effect from each variable respectively to establish unfavorable environmental conditions for dengue transmission. A modified version of the Einstein fuzzy operator is used to measure the combined effect from above factors. This operator overcomes the disadvantages of the Hamacher operator; another widely used fuzzy operator to combine two fuzzy membership values [12]. The Hamacher operator computes the combined effect to be zero if one individual membership value is zero no matter how much the other membership value is. For an example if the RF is totally favorable (membership = 0) while TEMP is slightly favorable (membership $>$ 0) then the Hamacher operator produces a membership value of zero which implies a totally favorable climate condition for dengue transmission and is sometimes misleading.

The value in [0, 1] computed by the new operator describes the overall effect from climate to create an unfavorable environmental condition for dengue transmission and does not reflect the risk directly. Therefore we transform the value to have a potential risk measure by subtracting it from one. The dengue cases are standardized and they are transformed to a time series in [0, 1]. The error function is obtained by comparing the predicted risk by the fuzzy model and the risk described by the standardized real dengue cases. Here we argue that the predicted risk at time t is a potential risk measure to understand the real risk in a future time $t + j$ for any $j \in [0, T]$ where T is the size of the time series.

It is known that temparature and rainfall data are subject to measurement errors. If these data are not accurate then the fuzzy model may produce inaccurate results and they may lead to wrong conclusions. Therefore a sensitivity analysis of the model is carried out to investigate how the results change in response to small variations in the climate parameters. We assume that these measurement errors of the climate parameters are random process each of them are uniformly distributed in fairly small intervals. This measurement errors are additively included. Again the a accuracy of the the fuzzy risk measure is investigated.

2 METHODOLOGY

2.1 Mathematical Preliminaries

Definition 1. Let U be a non-empty set and A, a subset of U. The characteristic function of A is given by

$$A(x) = \begin{cases} 1, & \text{if } x \in A; \\ 0, & \text{if } x \notin A; \end{cases} \tag{2.1}$$

Definition 2. A fuzzy subset F of U is described by the function $F : U \to [0, 1]$ called the membership function of fuzzy set F where U is a classical non-empty set.

The value $F(x) \in [0, 1]$ indicates the membership degree of the element x of U in fuzzy set F, with $F(x) = 1$ and $F(x) = 0$ representing, respectively, the belongingness and not-belongingness of x in F [8, 13].

Definition 3. If A is a fuzzy subset of X then the α-cut is defined as the non-fuzzy subset such that [14]

$$A_\alpha = \{x | U_\alpha(x) \geq \alpha\} \text{ for } 0 < \alpha \leq 1 \tag{2.2}$$

Definition 4. The intersection of A and B, denoted $A \cap B$, is defined on the largest fuzzy set contained in both A and B, given by the membership function

$$U_{A\cap B}(x) = \min\{U_A(x), U_B(x)\} \text{ for each } x \in F. \tag{2.3}$$

The union of A and B, denoted $A \cup B$, is defined on the smallest fuzzy set contained in both A and B,given by the membership function

$$U_{A\cup B}(x) = \max\{U_A(x), U_B(x)\} \text{ for each } x \in F. \tag{2.4}$$

The Einstein sum is defined as [15]

$$U_{ES}(x) = \frac{U_A(x) + U_B(x)}{1 + U_A(x) \cdot U_B(x)}. \tag{2.5}$$

Definition 5. A fuzzy set is concentrated by reducing the grade of membership of all elements that are only partly in the set, in such a way that the less an element is in the set, the more its grade of membership is reduced. The concentration of a fuzzy set A can be defined by [14]

$$U_{CONC(A)}(x) = U_A^a(x) \text{ with } a > 1. \tag{2.6}$$

The opposite of the concentration is the dilation. A fuzzy set is dilated or stretched by increasing the grade of membership of all elements that are partly in the set. The dilation of a fuzzy set A can be defined by [14]

$$U_{DIL(A)}(x) = U_A^a(x) \text{ with } a < 1. \tag{2.7}$$

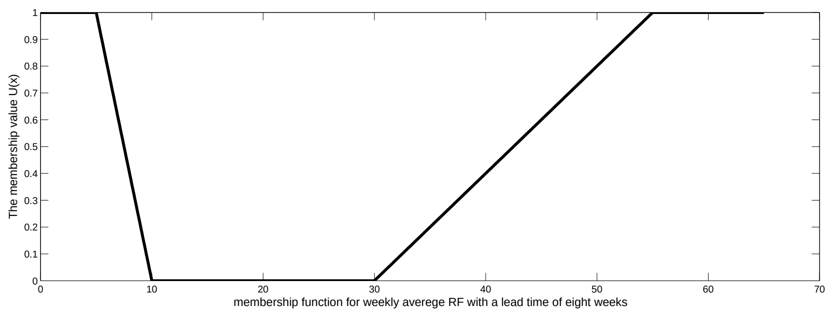
2.2 Mathematical Models

We assume that at least 5mm averaged weekly rainfall is required to make breeding sites available for mosquitoes and the breeding sites are washed out due to the heavy rainfall which is over 55mm [5, 16, 17]. Further it is assumed that a weekly average temperature less than 16^0C is unfavorable for mosquitoes to transmit the virus and a temperature between 30^0C and 34^0C is ideal for mosquitoes to rapidly transmit of the virus due to the increased vector capacity and reduced incubation period [18]. According to literature extreme heating conditions do not support dengue virus transmission so that we assume the threshold temperature to be 37^0C [5, 16, 18]. Based on these conditions, the trapezoidal-shaped membership functions $U_{RF}(x) : A \subseteq \mathbb{R} \to [0,1]$ and $U_{RF}(x) : B \subseteq \mathbb{R} \to [0,1]$ are defined respectively to represent the effect from eight weeks leading RF and immediate TEMP to create an unfavorable environment for dengue as

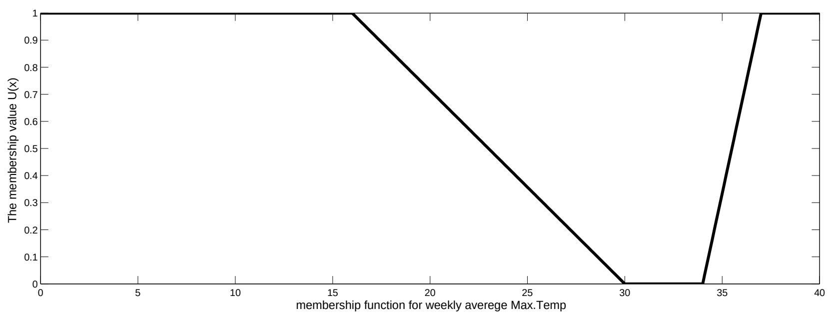
$$U_{RF}(x) = \begin{cases} 1, & \text{if } x \leq 5; \\ -\frac{x+10}{5}, & \text{if } 5 \leq x \leq 10; \\ 0, & \text{if } 10 \leq x \leq 30; \\ \frac{x-30}{25}, & \text{if } 30 \leq x \leq 55; \\ 1, & \text{if } x \geq 55; \end{cases} \tag{2.8}$$

$$U_{TEMP}(x) = \begin{cases} 1, & \text{if } x \leq 16; \\ -\frac{x+30}{14}, & \text{if } 16 \leq x \leq 30; \\ 0, & \text{if } 30 \leq x \leq 34; \\ \frac{x-34}{3}, & \text{if } 34 \leq x \leq 37; \\ 1, & \text{if } x \geq 37; \end{cases} \tag{2.9}$$

The trapezoidal-shaped membership functions given in (2.8) and (2.9) are illustrated in Fig 1a and Fig 1b respectively.



(a) Average weekly rainfall



(b) Average weekly maximum temperature

Fig. 1. Memebership functions

2.2.1 Fuzzy operators

It should be noted that the parameters rainfall and temperature considered separately do not provide any measure of climate risk for dengue transmission. Further the two parameters are in two different scales. Therefore we need to find a technique to combine these two parameters and get them into a single scale. The theory of fuzzy operators overcomes this problem. The methodology of transforming two climate parameters into fuzzy measure and obtaining a single risk measure is illustrated in Fig 2.

We define the modified operator which computes the overall effect as a function $U_{MES}(x) : ([0,1]\times[0,1]) \to [0,1]$ given by

$$U_{MES}(x) = \frac{U^2_{RF}(x) + U^2_{TEMP}(x)}{1 + U_{RF}(x) \cdot U_{TEMP}(x)}. \qquad (2.10)$$

It is obviously seen that,

- If $U_{RF}(x) = 0$ and $U_{TEMP}(x) = 0$ then $U_{MES}(x) = 0$.
- If $U_{RF}(x) = 0$ and $U_{TEMP}(x) \neq 0$ then $U_{MES}(x) = U^2_{TEMP}(x) \leq U_{TEMP}(x)$ or if $U_{TEMP}(x) = 0$ and $U_{RF}(x) \neq 0$ then $U_{MES}(x) = U^2_{RF}(x) \leq U_{RF}(x)$.

Further it can be shown that if $U_{TEMP}(x) < U_{RF}(x)$ then $U_{MES}(x) < U_{RF}$.
The overall climate risk with respect to different levels of rainfall and temperature are given in Fig 3.

Then we define the potential risk of dengue transmission again as a function $M : ([0,1]\times[0,1]) \to [0,1]$ given by

$$M(x) = 1 - U_{MES}(x). \qquad (2.11)$$

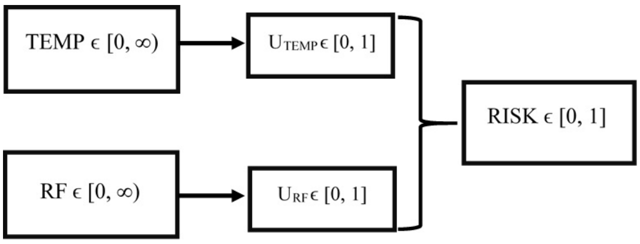


Fig. 2. The schematic diagram of transforming parameters into a single risk measure

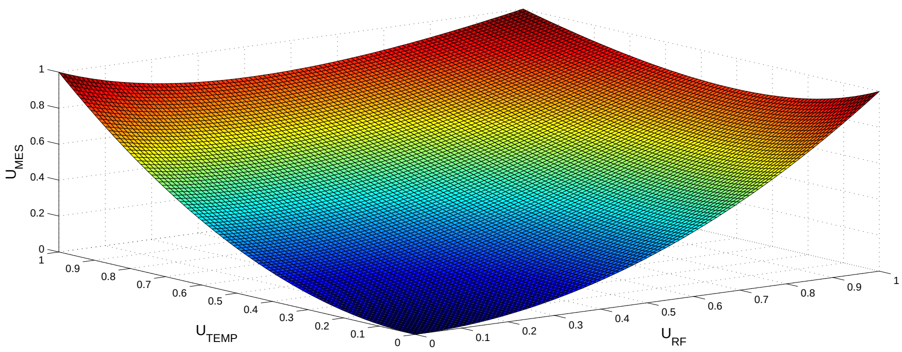


Fig. 3. The overall effect of climate risk for dengue transmission

3 DATA AND ANALYSIS

We use daily rainfall and maximum temperature data in urban Colombo from year 2006 to 2015 which obtained from the department of meteorology and transformed into average weekly data for the above two parameters. The weekly dengue cases data are standardized in such a way that each number of weekly dengue cases in a particular year is divided by the maximum number of weekly dengue cases incurred in the same year. The standardized dengue cases are identified as the real risk of dengue transmission denoted by $D_t^s \in [0, 1]$.

3.1 Error Analysis

It is understood that the predicted risk at time t should be realted to the real dengue risk reflected by dengue cases in a future interval of time $[t, t + j]$ for any $j \in [0, T]$ where T is the size of the time series. We define the mapping from predicted potential risk to a cluster of real risk of dengue as $f_t : [0, 1] \to [0, 1]_{[t,t+j]}$ for any $j \in [0, T]$ and given by

$$\begin{aligned} f_t(M_t) &= D^s_{t'\in[t,t+j]} \\ \text{where } |D^s_{t'} - M_t| &= inf|D^s_t - M_t| \\ \text{for any } t, t' &\in [t, t + j]. \end{aligned} \tag{3.1}$$

The error function is defined as

$$e_t = |f_t(M_t) - M_t|. \tag{3.2}$$

Dengue is an epidemic disease attributed with a complex transmission dynamics. A model which predicts the dengue risk for a level of accuracy of $e_t \leq 0.1$ would be totally effective to establish an early warning system. Based on the cluster of real risk of dengue, the level of accuracy of the model A_t is defined as follows.

$$A_t = \begin{cases} 100\%, & \text{if } e_t \leq 0.1; \\ 90\%, & \text{if } 0.1 < e_t \leq 0.25; \\ 80\%, & \text{if } 0.25 < e_t \leq 0.4; \\ 50\%, & \text{if } e_t > 0.4; \end{cases} \tag{3.3}$$

4 SENSITIVITY ANALYSIS

Climate data are subject to measurement errors. Therefore it is very important to investigate how our fuzzy model for climate risk responses to these errors. The impact of the measurement errors in the climate parameters should not be significantly large for the final results from the model. We carry out a sensitivity analysis to investigate how the fuzzy model reacts to a small change in the parameters.

It should be noted that the measurement errors of the climate parameters are not fixed with respect to time. We assume they are uniformly distributed random variables. Now we adjust our two parameters such that

$$RF_{adj}(t) = \max[0, RF(t) + \delta_1(t)] \text{ and}$$
$$TEMP_{adj}(t) = TEMP(t) + \delta_2(t)$$

where $\delta_1(t) \sim uni[-2, 2]$ and $\delta_2(t) \sim uni[-0.5, 0.5]$ for each t. Here we assume that the rainfall is measured to the nearest 2mm and the temperature is measured to the nearest 0.5 degrees of celsius. The adjusted parameters are also transformed to fuzzy measures in $[0, 1]$ using the same fuzzy membership functions given in (2.8) and (2.9). Again the accuracy of the model to predict the real risk of dengue transmission in urban Colombo is investigated using the algorithm introduced in section **??**.

5 RESULTS AND DISCUSSION

The dengue cases time series from year 2006-2015 is presented in Fig 4. The trend in the dengue cases data is removed using the methodology described early in this paper and the resulted time series is given in Fig 5 which gives the real risk of dengue from year 2006-2015. The predicted potential risk of dengue transmission from RF and TEMP is computed using the Modified Einstein Sum operator and its behavior is shown in Fig 6. The error variation between the real risk and the predicted potential risk is illustrated in Fig 7.

The real risk time series seems to be highly non stationary as it is expected that the epidemiological data are noisy due to its complexity. The predicted potential risk time series attains to either 0 or 1 number of times and this implies for certain time points the risk is very high or no risk at all.

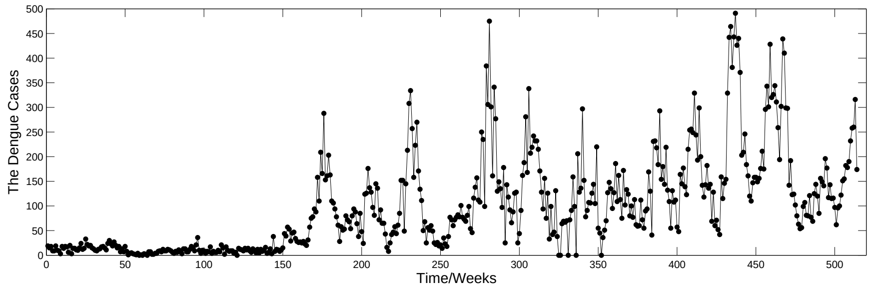


Fig. 4. The weekly dengue cases distribution from year 2006-2015

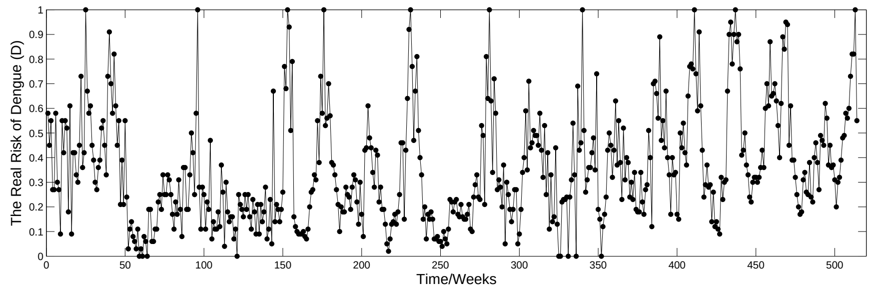


Fig. 5. The real risk of dengue cases distribution from year 2006-2015

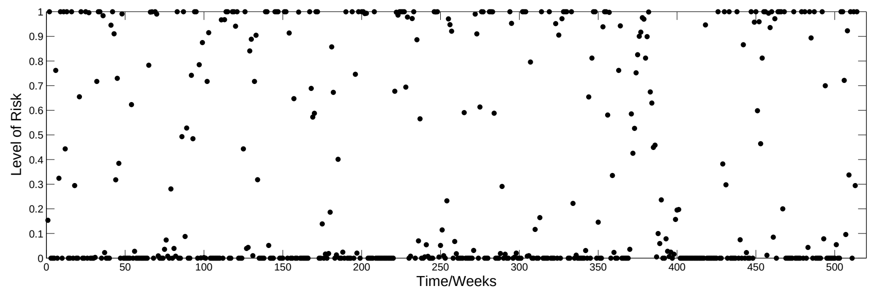


Fig. 6. The predicted potential risk distribution of dengue transmission from year 2006-2015

We argue that the risk predicted from the model for time t can be used to have an idea about the real dengue risk for further j time periods, that is up to time $t + j$. We set j equals to approximately 2 months. This method is useful in epidemic modeling since the predicted risk in a certian time must be taken to implement control strategies in a future time. Based on this mapping algorithm, the accuracy of our model is investigated. It is observed from the analysis that, 42.80% of the time the level of accuracy is 100 percent, 26.46% of the time the level of accuracy is 90 percent and 16.15% of the time it is 80 percent. Generally the level of accuracy of the model is more than 80 percent in 86.77% of the time considered in the study.

We carry out the sensitivity analysis of the model to investigate how the system responses to a small change in the climate parameters according to the method described in section **??**. The sensitivity analysis suggests that 44.75% of the time the level of accuracy is 100 percent. Generally the level of accuracy of the model is more than 80 percent in 87.55% of the time considered in the study. These results suggest that the model responses well to the small changes in the values of the two climate parameters. Thus the measurement errors do not have a significant impact on the model outcomes. The accracy of the model after the sensitivity analysis is given in Fig 8.

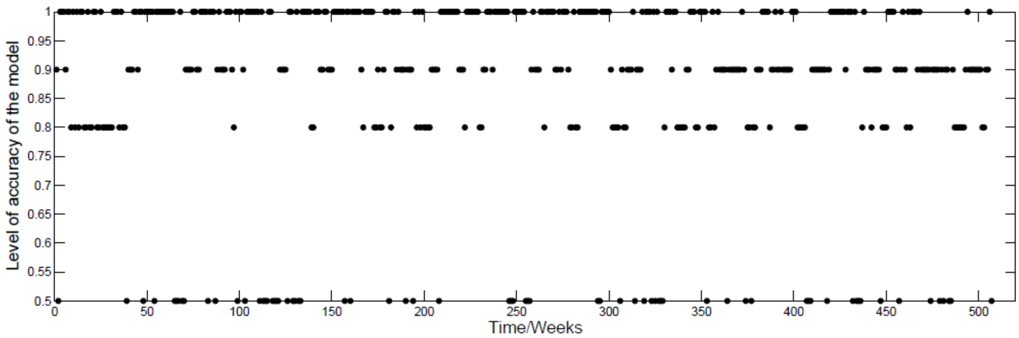


Fig. 7. The level of accuracy of the model from year 2006-2015

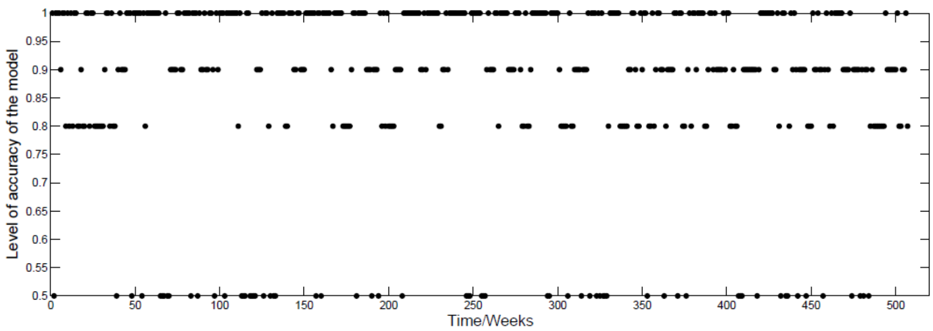


Fig. 8. The sensitivity of the model

6 CONCLUSIONS

Fuzzy set theory can be highly useful when modeling and analyzing uncertain, complex systems such as disease spread. Theory of fuzzy operators can be used to investigate the overall potential risk of dengue disease transmission from several sensitive parameters such as rainfall and temperature. The overall effect from rainfall and temperature to produce unfavorable environmental conditions for dengue transmission is computed in the first phase using the modified Einstein Sum operator. The predicted potential risk is evaluated by subtracting this measure of unfavorability from one. The standardized dengue cases are identified as the real risk of dengue transmission and the accuracy of the model is investigated using the algorithm described in the data and analysis section. However we observe that the predicted potential risk is overestimated for certain weeks and it is underestimated for some weeks. The underestimation of the predicted risk might be due to the fact that, we only considered the influence of rainfall and temperature in the model. Human mobility, demography and geography are other vital factors which are sensitive for the dynamics of dengue disease transmission. We aim at modeling the risk of dengue transmission with respect to climate and this risk is implicitly described by the mosquito density and its spread. Eventhough a some level of risk is predicted by the model, the real dengue cases do not reflect the actual risk may be because people use various techniques to avoid mosquito contacts by using insecticides, bed nets and etc. However we observe from the analysis that the model is more than 80 percent accurate in 86.77% of the time. A sensitivity analysis is carried out to investigate the impact of the measurement errors to the outcomes og the fuzzy model of climate risk. The analysis suggests that the model responses well to these errors in the data collection.

Dengue is an epidemic disease with an extremely complicated transmission process. Large number of various factors influencing its transmission in different levels in different times. A neutral factor in a particular period of time can be dominant in a different time period. An expanded model based on the methodology discussed in this paper with the consideration of other factors such as human mobility, demography and geography can be useful in predicting the risk even more accurately.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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